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SPATIAL LINEAR APPROXIMATE TO ALMOST IDEAL DEMAND SYSTEM

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Abstract

The paper introduces two spatial systems of demand equation models that are variants of the Linear Approximate to Almost Ideal Demand System (LA/AIDS) with habit formation. Specifically, the first model incorporates spatial lag of the quantity demanded in the LA/AIDS and the second model incorporates spatial lag of budget shares. An empirical application using fish expenditure allocation data from the Philippines showed that the two models performed better compared to the static LA/AIDS.

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1. Introduction

In our previous paper [8], we estimated single spatial aggregate fresh fish demand functions that take into account the spatial dependence of the data. We showed that in general, a spatial lag *aka* spatial autoregressive (SAR) model specification that contains spatial lag of the dependent variable revealed some important information that will escape notice if such spatial dependence is ignored. Our SAR demand model is analogous to the habit formation model (HFM) [6] for the case of time series data. In the same paper, we specified and estimated aggregate fresh fish demand function using the same data set that allows elasticities of demand to vary across the country taking into account spatial dependence. Combined with cluster analysis and with the aid of geographic information system, our results revealed distinct clusters of locations which are characterized by different consumption behavior.

This paper is an extension of our previous paper mentioned above in a system of demand equations framework. In particular, we introduce two spatial systems of demand equations that are variants of the Linear Approximate to Almost Ideal Demand System (LA/AIDS) model [4]. The first model incorporates spatial lag of quantity demanded in the LA/AIDS which is a resemblance of the LA/AIDS with habit formation in time series case as in Chen and Veenam [3] and Lariviere et al. [7]. The second model incorporates spatial lag of budget shares (the dependent variable) which is a resemblance of our previous single equation SAR model in [8]. To evaluate the performance of these two models, we applied them using the same data from the Philippines and compared the results to that of the static LA/AIDS. Results showed that the spatial models performed better than the LA/AIDS.

The paper is arranged as follows. Following this introduction is a brief overview of demand models with particular emphasis to FHM and LA/AIDS. Section 3 formally introduces the spatial models and their inherent properties. Section 4 presents the empirical application and the final section (Section 5) provides conclusion and recommendations for further research.

2. Demand Models

2.1. Single equation and the habit formation model

The first empirical demand studies specified single equation demand functions linear in the parameters and quantity dependent, of which the double log (i.e., Cobb-Douglas) was the most common specification. These functions are either specified using a time series or cross-section data. Letting q_{it} denote the quantity consumed of good i at time t, with corresponding price p_{it} and X_t be the total expenditure/budget, the equation to be estimated is:

$$\ln q_{it} = \alpha_{0i} + \sum_j \alpha_{ij} \ln p_{jt} + \beta_i \ln X_t. \quad (1)$$

By virtue of the log transformation, we can interpret the estimated parameters as elasticities as:

$$\alpha_{ij} = \partial \ln q_{it} / \partial \ln p_{jt}, \quad (2)$$

$$\beta_i = \partial \ln q_{it} / \partial \ln X_t. \quad (3)$$

Specifically, (2) pertains to the price elasticities and (3) to the expenditure elasticity. The range of j varies, and typically includes commodities that are assumed to be closely associated with good i.

During the 1960's and 70's, dynamic demand models are introduced such as the HFM [6]. The dynamics are introduced in the lagged consumption variable, q_{it-1} which makes current consumption dependent on the previous period's consumption. Mathematically,

$$\ln q_{it} = \alpha_{0i} + a \ln q_{it-1} + \sum_j \alpha_{ij} \ln p_{jt} + \beta_i \ln X_t. \quad (4)$$

The long run elasticities may then be computed as:

$$\eta_{ij} = \alpha_{ij}(1 - a_i)^{-1} \quad (5)$$

and

$$\eta_i = \beta_i(1 - a_i)^{-1}. \quad (6)$$

2.2. System of demand equations and the LA/AIDS model

Single equation demand models were criticized for lack of theoretical consistency. The double log defined in (1) is theoretically consistent only when demand is independent of expenditure, i.e., the consumer's preferences are homothetic. This gave rise to the specification of systems of demand equations derived explicitly from consumer theory (See [5] for detailed discussion on consumer theory). Among these models, the AIDS (Almost Ideal Demand System) model is perhaps the most popular functional form. The AIDS is formulated in terms of budget shares (w_i). The static AIDS is of the form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P), \tag{7}$$

where P is a price index defined by:

$$\ln P = a_0 + \sum_j a_j \ln p_j + 1/2 \cdot \sum_i \sum_j b_{ij} \ln p_i \cdot \ln p_j. \tag{8}$$

The AIDS is linear except for the translog price P. A linear approximation (LA/AIDS) was proposed [4] by replacing P by a Stone price index (P^*):

$$P^* = \exp\left(\sum_i w_i \cdot \ln p_i\right). \tag{9}$$

The constraints of demand theory imply that:

$$\sum_i \alpha_i = 1, \ \sum_i \gamma_{ij} = 0, \ \sum_i \beta_i = 0, \tag{10}$$

$$\sum_j \gamma_{ij} = 0 \tag{11}$$

and

$$\gamma_{ij} = \gamma_{ji}. \tag{12}$$

In particular, (10) refers to the adding-up restriction, (11) refers to the homogeneity restriction and (12) to the symmetry restriction.

The elasticities are given by:

Expenditure elasticity:

$$\eta_i = 1 + \beta_i / w_i. \tag{13}$$

Uncompensated price elasticities:

$$\varepsilon_{ij} = [\gamma_{ij} - (\beta_i \cdot w_j)]/w_i - \delta_{ij}. \tag{14}$$

Compensated price elasticities

$$\varepsilon_{ij}^{c} = \varepsilon_{ii} + \eta_i w_i, \tag{15}$$

where δ_{ij} is the Kronecker delta which is equal to 1 if $i = j$ and 0 otherwise.

2.3. Habit formation variables in LA/AIDS

In a time series data, Chen and Veenam [3] incorporated consumption habit formation into the AIDS model adopting the "dynamic translating" procedure of [9] and [10]. The resulting model replaces α_i in (6) by the linear dynamic translating parameter $\alpha_i = \alpha_i^* + aq_{it-1}$:

$$w_i = \alpha_i^* + a_i q_{it-1} + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P), \tag{16}$$

$$\ln P = a_0 + \sum_j (\alpha_i^* + a_i q_{it-1}) \ln p_j + 1/2 \cdot \sum_i \sum_j b_{ij} \ln p_i \cdot \ln p_j. \tag{17}$$

Consequently, the uncompensated price elasticities are computed using the formula:

$$\varepsilon_{ij} = \left[\gamma_{ij} - \beta_i \left(\alpha_i^* + a_i q_{it-1} + \sum_j \gamma_{ij} \ln p_j \right) \right] \Big/ w_i - \delta_{ij}, \tag{18}$$

while the expenditure elasticity formula remained the same, (i.e., (13)).

The adding up condition requires:

$$\sum_i \alpha_i^* = 1, \ \sum_i \gamma_{ij} = \sum_i \beta_i = \sum_i a_i q_{it-1} = 0. \tag{19}$$

The restriction $\sum a_i q_{it-1} = 0$ requires that at least one of the a_i is negative. A positive sign indicates persistence and a negative sign implies negative inflation effects [7]. The conditions of homogeneity and symmetry remained the same, (i.e., (11) and (12)).

Recently, Lariviere et al. [7] augmented the specification of the LA/AIDS with habit formation variables that include lagged quantities of all the commodities, time trend (t) and trigonometric variables to capture seasonal and advertising effects. Mathematically, the model takes the form:

$$w_i = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P) + \sum_j \Phi_{ij} g(A_{jt}) + \sum_j \Omega_{ij} q_{jt-12}$$

$$+ \sum_{k=1}^{6} (k_{cik} \cdot \cos\phi_k + k_{sik} \cdot \sin\phi_k) + \varphi_i t, \tag{20}$$

where $\phi_k = 2\pi k$, $k = 1, 2, \ldots, 6$ and gA_{jt} captures the advertising effects. The restrictions and the computation of elasticities follow that of the original LA/AIDS specification as provided in (10-12) and (13-15), respectively.

3. Spatial LA/AIDS: Specification

We introduce in this section our spatial versions of the LA/AIDS that augment spatial lagged variables for lattice data. Specifically, we specified (i) a spatial LA/AIDS version of equation (16), and (ii) a spatial system of demand equation version of the original HFM of Houthakker and Taylor defined in (4). The spatial lags are operationalized by standardized spatial weight matrix (W). We refer to the first model as the LA/AIDS with spatial habit formation or SLA/AIDS-HFM and the second model as the Spatial Autocorrelated LA/AIDS or the SAR-LA/AIDS.

Denote l as subscript for the lth location, the SLA/AIDS-HFM can be specified as:

$$w_i = \alpha_i^* + a_i W q_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P), \tag{21}$$

where Wq_i is the spatial lag of q_i which is the average of q_i across the neighborhood of location l. That is:

$$Wq_i = \sum_{j=1}^{k} \omega_{lm} \cdot q_{ij}, \tag{22}$$

where k is the number of predefined neighboring locations of the focal location l, and ω_{lm}'s are the elements of the W matrix which define the neighborhood structure of the locations.

Similarly, the SAR-LA/AIDS can be specified as:

$$w_i = \alpha_i^* + a_i W w_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln(x/P). \tag{23}$$

Simply put, the models state that the consumption of commodity i in a particular location is dependent on the level of consumption in neighboring locations. The intuition behind this is that spatially adjacent regions exhibit similar consumption behavior due to inherent common spatial influences such as weather, regional market influences, regional production similarities, etc.

To be consistent with the fundamental postulates of demand theory, the adding up, symmetry and homogeneity conditions defined in (10-12) must hold in terms of parameter restrictions. The spatial lag parameters a_i must be between zero and one and this can be empirically observed. A negative a_i indicates that neighboring locations exhibit more dissimilar relationships than distant locations, a result counter to intuition. The elasticities can be easily computed following (13-15).

4. Application

4.1. Data and estimation procedure

This section illustrates the SLA/AIDS-HFM and SAR-LA/AIDS to fish expenditure allocation using data from the Philippines [8]. The models identified 11 fish types comprising of 8 fresh and 3 processed fish. A descriptive summary of these fish types is provided in Table 1.

Table 1. Summary statistics

Fish types	Consumption (q) (kg/capita/yr)	Price (P) (PhP/kg)	Expenditure Share (w_i)
Fresh fish			
1) Tilapia	2.71	56.55	0.11
2) Roundscad	3.78	54.62	0.13
3) Anchovy	1.05	50.44	0.03
4) Milkfish	2.39	82.04	0.13
5) Squid	0.61	80.94	0.03
6) Shell	0.51	57.11	0.02
7) Shrimp	0.36	170.64	0.04
8) Other fresh fish	8.08	66.08	0.29
Processed fish			
9) Canned fish	8.03	15.25	0.08
10) Smoked fish	2.17	86.47	0.11
11) Salted fish	2.46	17.06	0.02

Note: 1US$ = PhP 40.55

Both models were estimated at province-level [8] using the ITSUR (Iterative Seemingly Unrelated Regression) method of the SYSLIN of SAS (Statistical Analysis System) [11]. In the process, an error term must be added in all the equations. As in the usual AIDS system of equations estimation, one of the equations is deleted (in this case salted fish) to avoid singularity (i.e., budget shares sum-up to 1). The parameters of the omitted equation can be recovered by virtue of the adding up restriction. The homogeneity and symmetry restrictions are imposed econometrically during the estimation. Following our previous paper [8], the (W) was specified using the $(k = 6)$ nearest neighbor relation.

As shown in Table 1, roundscad, milkfish and tilapia are the most popular fish species in the Philippines. These three fish commodities account almost 40% of the total fish expenditure. Among these three, milkfish (a brackishwater fish) is relatively more expensive, while the price of roundscad (a marine fish) and tilapia (culture fish) are more or less the same. Roundscad is actually regarded as "poor mans' fish". Per capita consumption of the different fish commodities vary across the country. Figure 1 depicts the distribution of per capita consumption for roundscad and milkfish, where some pattern of regionalization can be deduced.

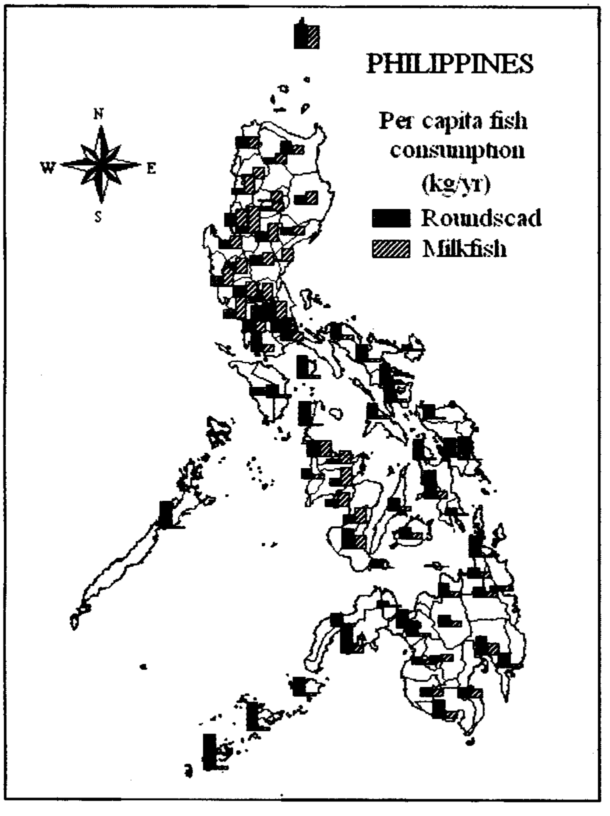

Figure 1. Distribution of per capita consumption of roundscad and milkfish, Philippines, 2000.

4.2. Results

Figure 2 compares the single-equation[1] adjusted- and the system-R^2 of the LA/AIDS, SLA/AIDS and SAR-LA/AIDS. It can be concluded that the spatial models have better fit compared to the traditional LA/AIDS. The system-R^2 of SAR-LA/AIDS is 0.48 compared to 0.34 of SLA/AIDS – HFM and 0.25 of LA/AIDS. A system-R^2 of 0.48 of the SAR-LAIDS indicates that the whole system explains 48% of the total variation in fish expenditure allocation.

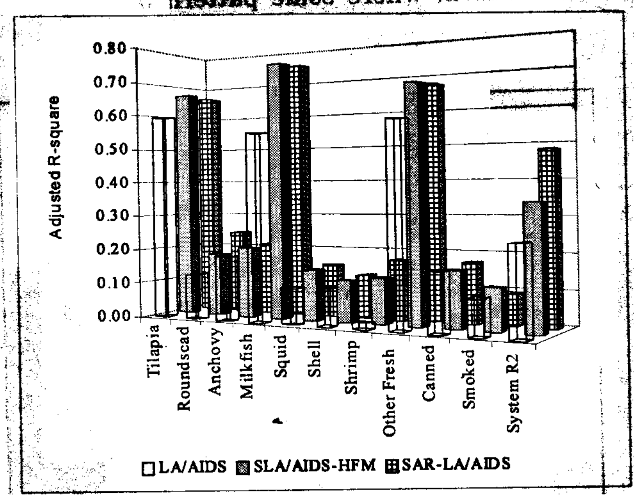

Figure 2. Comparison of the single-equation adjusted- and system-R^2.

Table 2 compares the estimated spatial lag parameters (a_i) of SLA/LAIDS-HFM and SAR-LA/AIDS. All the a_i's exhibit positive sign and the joint tests are significant at 99%. In particular, the a_i's of SAR-LA/AIDS which can be interpreted as spatial autocorrelation are all significant with values ranging from 0.40 (shell) to 0.79 (squid). This indicates clustering of provinces with high budget shares for a particular fish types.

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[1] Though single-equation R^2 statistics are not good measure of fit for a system of equations ([11] and [2]), it is interesting to note how the SRM and LA/AIDS differ in terms of these statistics.

Table 2. Comparison of the estimated spatial lag (a_i) parameters

	SLA/AIDS-HFM		SAR-LA/AIDS	
	a_i	s.e.	a_i	s.e.
Tilapia	0.009	0.002	0.730 ***	0.078
Roundscad	0.004*	0.002	0.739 ***	0.079
Anchovy	0.006*	0.003	0.773 ***	0.147
Milkfish	0.001	0.002	0.730 ***	0.067
Squid	0.009*	0.005	0.788 ***	0.198
Shell	0.005	0.004	0.403 *	0.210
Shrimp	0.032**	0.012	0.737 ***	0.131
Other fresh fish	0.003***	0.001	0.720 ***	0.062
Canned fish	0.003*	0.002	0.498 ***	0.165
Smoked fish	0.000	0.002	0.708 ***	0.086
	***			
Joint test for $a_1 = a_2 =, ..., = a_{10} = 0$				
F-value	8.00***		22.64 ***	

* Significant at $\alpha = 0.10$

** Significant at $\alpha = 0.05$

*** Significant at $\alpha = 0.01$

The relatively lower in magnitude of the a_i's of SLA/AIDS-HFM and the non-significance of some suggest that in these clusters of provinces, a high quantity demanded of a particular fish type does not necessarily follow that the budget share relative to other species is also high. Thus, the spatial dimension can be best represented by spatial autocorrelation, i.e., spatial lag of budget shares.

Table 3 compares the expenditure elasticity (η_i) of the three systems. It can be seen that the η_i of LA/AIDS and SLA/AIDS-HFM exhibit similar pattern. There are 3 indications to support that the η_i of SAR-LA/AIDS are better than that of the LA/AIDS and SLA/AIDS-HFM. First, LA/AIDS and SLA/AIDS-HFM indicate that anchovy which is the cheapest among the fresh fish commodities (see Table 1) is a luxury commodity ($\eta_i > 1$)- a result that is questionable. Second, LA/AIDS and SLA/AIDS-HFM indicate that tilapia is an inferior good ($\eta_i < 0$) which is on average highly unlikely in the Philippines. In fact, one would expect that milkfish, tilapia and roundscad would have more or less similar η_i (see Table 1). This leads us to the third indication to support that SAR-LA/AIDS's η_i are better. The η_i of milkfish LA/AIDS and SLA/AIDS-HFM are 0.14 and 0.13, respectively compared 0.82 and 0.77 of roundscad. On the other hand, the SAR-LA/AIDS posted a η_i for 0.57 which is closed to 0.70 for roundscad.

Table 3. Comparison of the estimated expenditure elasticities (η_i)

	LA/AIDS	SLA/AIDS-HFM	SAR-LA/AIDS
Tilapia	–0.49	–0.44	0.09
Roundscad	0.82	0.77	0.70
Anchovy	1.33	1.25	0.89
Milkfish	0.14	0.13	0.57
Squid	1.58	1.44	1.28
Shell	0.44	0.49	0.49
Shrimp	0.85	0.79	0.74
Other fresh fish	2.27	2.28	1.88
Canned fish	0.48	0.49	0.55
Smoked fish	1.01	0.98	1.06

Table 4 compares the compensated own-price elasticities (ε_{ii}^{c}) of the three systems. Again, LA/AIDS and SLA/AIDS-HFM exhibit similar pattern. A point to note is that roundscad is elastic ($\varepsilon_{ii}^{c} > 1$) so with tilapia and milkfish under SAR-LA/AIDS while it is almost unitary elastic ($\varepsilon_{ii}^{c} \approx 1$) under LA/AIDS and SLA/AIDS-HFM which is very different from tilapia.

Table 4. Comparison of the estimated own-price elasticities

	LA/AIDS	SLA/AIDS-HFM	SAR-LA/AIDS
Tilapia	–1.53	–1.35	–1.21
Roundscad	–1.02	–1.07	–1.23
Anchovy	–1.23	–1.15	–1.36
Milkfish	–2.24	–2.35	–1.47
Squid	–1.08	–0.99	–1.04
Shell	–1.08	–1.03	–0.95
Shrimp	–1.05	–1.02	–1.03
Other fresh fish	–1.78	–1.72	–1.08
Canned fish	–0.44	–0.39	–0.43
Smoked fish	–1.68	–1.67	–1.62

5. Conclusion and Recommendation

The paper extends our previous study on incorporating spatial dimension on demand models into a system of equation framework. Two spatial systems of demand equation models that are variants of the LA/AIDS are introduced. The first model which is regarded as the SLA/AIDS-HFM incorporates spatial lag of quantity demanded in the LA/AIDS that resembles the LA/AIDS with habit formation. The second model which is regarded as SAR-LA/AIDS incorporates spatial lag of the budget share (dependent variable) which resembles our single equation SAR model and that of the original single-equation habit formation model. An empirical application using fish expenditure allocation data from the Philippines showed that the two models performed better compared to the static LA/AIDS with the SAR-LA/AIDS model explaining relatively higher proportion of the variance of fish expenditure allocation and producing more sensible elasticities.

In the literature of demand analysis, dynamic system of equation models specified in the first difference form such as the Rotterdam model ([1] and [12]) are often used. Therefore, a topic for further research would be to specify spatial system of demand equations that are resemblance of the Rotterdam and the first difference AIDS models.

Lastly, as shown in the paper, consumption of fish by fish types varies significantly across the country. Thus, another topic for further research would be to specify spatial system of demand equations that will capture the spatial instability of consumption behavior.

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