An Empirical Comparison of Seasonal Growth Models

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Abstract

A model for describing the seasonally oscillating growth of fish is presented and compared with the commonly used model of D. Pauly and G. Gaschütz (PG). The new model produces results similar to those obtained from the PG model, except for the parameter t_o which the new model estimates accurately, such that $L_t = 0$ when $t = t_o$.

Introduction

Several authors (Beverton and Holt 1957; Ursin 1963; Lockwood 1974; Pitcher and MacDonald 1973; Cloern and Nichols 1978; Pauly and Gaschütz 1979) have proposed extensions to the basic von Bertalanffy growth equation by introducing a seasonality component in the model. The last three models are commonly used to fit the growth pattern of fishes and so our study deals with them.

The Cloern and Nichols (CN) and Pitcher and MacDonald (PM) models were obtained through a differential equation approach, but it was not so in the case of Pauly and Gaschütz (PG) model. However, Pauly and Gaschütz have noted that the parameters in their model were more easily interpretable than those in the other two models. Therefore, a new model which is a combination of the features of PG and CN models was derived and subsequently compared with the others (Hoenig and Hanumara 1982). More recently, Somers (1988) obtained independently the new model in the same way as us. Hence, in this paper, we limit ourselves to an empirical comparison of the models.

The PG model is

$$L_t = L_\infty\{1-e^{-[K(t-t_o) + \frac{KC}{2\pi}\mathrm{SIN}\,2\pi(t-t_s)]}\}$$

where L_t is the length at t years of age, L_∞ the asymptotic length, K a growth constant, t_o the age at zero length, t_s sets the beginning of the sine wave and C determines the amplitude of the oscillation.

The new model is

$$L_t = L_\infty\{1-e^{-[K(t-t_o) + \frac{KC}{2\pi}\mathrm{SIN}\,2\pi(t-t_s) - \frac{KC}{2\pi}\mathrm{SIN}\,2\pi(t_o-t_s)]}\}$$

where the parameters are defined as in PG model. The result of the additional term in the new model compared with the PG model is that when $t = t_o$, L_t is zero. Somers (1988) gives an implication of this aspect of the model.

The PM model is

$$L_t = L_\infty\{1-e^{-[K(t-t_o) + C\,\mathrm{SIN}\,2\pi(t-t_s)]}\}$$

The CN model is

$$L_t = L_M-(L_M-L_m)\{1-e^{-[A_1(t-t_o) + \frac{A_1(\mathrm{COS}2\pi(t_o+\theta) - \mathrm{COS}2\pi(t+\theta))}{2\pi}]}\}$$

where t_o is the time of recruitment at minimum size L_m, L_M is the maximum size, and A_1 and 0 are constants. The PM and CN models are modified so the time measurement is in years as in the other two models.

Fit of Models

Pauly and Gaschütz (1979) have suggested first to estimate L_∞ using a method such as a Ford-Walford plot and then linearize the model so the linear regression methodology can be used for fitting. The same approach may be used to fit the new model. Then, the solutions for estimates $\hat{K}$, $\hat{T}$ and $\hat{C}$ are obtained explicitly but an iterative scheme is needed to find $\hat{t}_0$. In the case of CN model, not only $\hat{t}_0$ has to be solved iteratively; a constraint on the linear regression parameter estimates is needed in order for the normal equations to be consistent. No iterative methods are necessary to obtain the estimates of parameters in the PM model.

From the above discussion, we note that the linearization of a model does not always lead to explicit solution for the estimates of the parameters. Further, the properties of the estimators need to be evaluated for inferential purposes and since the error term has to be multiplicative to the model, the variance structure for it is restrictive. Hanumara and Hoenig (1987) and Hoenig and Hanumara (1982) investigated non-linear versus linearized fit of the models. The results indicate that the direct fitting of the non-linear model offers flexibility in choosing the variance structure for the error term, which represents a slight improvement over fitting the linearized model. Further, there is no need to provide a preliminary estimate of L_∞ or L_M. The non-linear fit of the model is not as complicated as it used to be with the availability of efficient software - except that a careful choice of initial values for parameter estimates is needed. This may be overcome by choosing the estimates obtained from a linearized fit of the model as initial values (as obtained using, e.g., by using the method of Soriano and Pauly 1989). Hence, we report our findings based on the non-linear fit of the models.

The data sets on pout, emerald shiner and halfbeak (Pauly and Gaschütz 1979) and male and female sole (Miller and Wellings 1971) are used. The error term e which is now additive to the model is chosen to have a variance as a function of t such as σ^2, $\sigma^2\sqrt{t}$ and $\sigma^2 t$. The value of L_m in the CN model is chosen to be zero. All the calculations were done using the SAS (1982, 1985) software package.

Results and Discussion

The fit is good in all cases with coefficient of determination exceeding .99. For brevity, we present the results (Tables 1 and 2) for only two data sets: emerald shiner and female sole which have 16 and 73 observations, respectively. In some cases, the choice of $\sigma^2\sqrt{t}$ or $\sigma^2 t$ over σ^2 for the variance of the error term e showed a slight improvement in the precision of the estimates. However, for the sake of consistency, the results are given when the variance of e is chosen to be σ^2.

Table 1. Fit of various seaonal growth models to emerald shiner data.

Parameter	Estimate	Asymptotic Std. Error	Coeff. of Var.	Asymptotic Corr. Matrix of the Estimates					R^2
				Pauly and Gaschütz Model					
K	0.9045	0.1118	0.1236	1.0000	0.9011	0.1456	-0.6180	-0.9802	0.9995
t_o	-0.0862	0.0294	-0.3411		1.0000	0.1380	-0.65557	-0.8137	
C	1.3980	0.0710	0.0508			1.0000	-0.0135	-0.1542	
t_s	0.0952	0.0091	0.0956				1.0000	0.5769	
L_∞	11.0136	0.6314	0.0573					1.0000	
				New Model					
K	0.9045	0.1118	0.1236	1.0000	0.8113	0.1456	-0.6180	-0.9802	0.9995
t_o	0.0178	0.0112	0.6292		1.0000	0.3669	-0.3231	-0.7289	
C	1.3980	0.0710	0.0508			1.0000	-0.0135	-0.1542	
t_s	0.0952	0.0091	0.0956				1.0000	0.5769	
L_∞	11.0136	0.6314	0.0573					1.0000	
				Pitcher and MacDonald Model					
K	0.9080	0.1491	0.1642	1.0000	0.8891	0.9214	0.6423	-0.9798	0.9991
t_o	-0.0838	0.0380	-0.4535		1.000	0.8196	0.6943	-0.7972	
C	0.1944	0.0358	0.1842			1.0000	0.5713	-0.9078	
t_s	0.1127	0.0133	0.1180				1.0000	-0.5940	
L_∞	10.9432	0.8473	0.0774					1.000	
				Cloern and Nichols Model					
A_1	0.7518	0.2267	0.3015	1.0000	0.8813	0.6476	-0.9895		0.9982
t_o	-0.0225	0.0321	-1.4267		1.0000	0.4722	-0.8204		
θ	0.1502	0.0252	0.1678			1.0000	-0.6246		
L_M	12.0107	1.8631	0.1551				1.000		

Table 2. Fit of various seaonal growth models to female sole data.

Parameter	Estimate	Asymptotic Std. Error	Coeff. of Var.	Asymptotic Corr. Matrix of the Estimates					R^2
Pauly and Gaschütz Model									
K	0.3925	0.0240	.0.0611	1.0000	0.8235	0.1167	-0.0835	-0.9357	0.9980
t_0	0.0737	0.0545	0.7106		1.000	0.1654	-0.1086	-0.6465	
C	0.8127	0.2212	0.2722			1.000	0.0404	-0.0808	
T_s	0.4084	0.0416	0.1019				1.0000	0.0574	
L_∞	399.9355	7.8088	0.0195					1.000	
New Model									
K	0.3925	0.0240	0.0611	1.0000	0.6989	0.1167	-0.0835	-0.9357	0.9980
t_0	0.2013	0.0565	0.2807		1.0010	0.6298	0.0650	-0.5443	
C	0.8127	0.2212	0.2722			1.0000	0.0404	-0.0808	
t_s	0.4084	0.0416	0.1019				1.0000	0.0574	
L_∞	399.9355	7.8088	0.0195					1.0000	
Pitcher and MacDonald Model									
K	0.3966	0.0258	0.0651	1.0000	0.8227	-0.3272	-0.0932	-0.9353	0.9977
t_0	0.0866	0.0575	0.6640		1.0000	-0.3275	-0.1398	-0.6455	
C	-0.0509	0.0148	-0.2908			1.0000	0.0629	0.2788	
t_s	-0.2264	0.0467	-0.2063				1.0000	0.0655	
L_∞	398.6849	8.2464	0.0207					1.0000	
Cloern and Nichols Model									
A_1	0.3941	0.0236	0.0606	1.0000	0.7962	0.0888	-0.9354		0.9980
t_o	0.2273	0.0380	0.1672		1.0000	-0.1416	-0.6269		
θ	1.8104	0.0336	0.0183			1.0000	-0.0608		
L_M	399.6224	7.7407	0.0194				1.000		

A comparison of the results for the PG and the new models shows that the estimates of parameters $\hat{K}$, $\hat{T}_s$, $\hat{C}$ and $\hat{L}_\infty$ and their standard errors are the same for both models (except for differences in the fifth decimal place) but differences from a low of 10% to more than 100% (considering all the data sets) occur in the estimates of $\hat{t}_0$. For the large data set, the coefficient of variation of $\hat{t}_0$ is smaller with the new model than with the PG model and the situation is reversed when using the small data set. However, no general conclusions can be drawn regarding the coefficient of variation of $\hat{t}_0$. The correlations of $\hat{t}_0$ with C and t_s are most affected when the new model is used.

The estimates of $\hat{K}$, $\hat{t}_0$ and $\hat{L}_\infty$ in the PM model are nearly the same as those in the PG model but that a large difference occurs in the estimate of $\hat{C}$. The coefficients of variation of all the estimators in the PM model are large in comparison with those of the PG or the new model. For female sole data, the estimates $\hat{A}_1$, $\hat{t}_0$ and $\hat{L}_M$ in the CN model match with and $\hat{K}$, $\hat{t}_0$ and $\hat{L}_\infty$, respectively.

The new model, derived using a differential equation approach presented in this paper, is probably a better choice than the PG model to fit seasonal growth data in fisheries. For moderately large data sets, the estimated parameters in the models remain very close to those given by the PG model, except for the parameter t_0.

Acknowledgments

We thank Prof. S. Saila for introducing us to this area of research. Our thanks also go to Nina Kajiji for checking the computations.

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