

**LENGTH-CONVERTED CATCH CURVES:  
A POWERFUL TOOL FOR FISHERIES  
RESEARCH IN THE TROPICS (PART I)**

by

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### Introduction

A number of methods are available with the help of which total mortality ( $Z$ ) can be estimated from length-frequency data. Thus, it is possible to obtain reasonable estimates of  $Z$  from the mean length in a representative sample (Beverton & Holt 1956), or from the slope of Jones' (1981) cumulative plot. These, and other methods for the estimation of  $Z$  are reviewed in Pauly (in press).

In the following, a variety of approaches for analysing length-frequency data are presented which represent the functional equivalent of [age structured] catch curves; these "length-converted catch curves", as I've chosen to name them, are built around assumptions similar to those involved in age-structured catch curves. However, the wider availability and the ease in collection of length-frequency data in the tropics and elsewhere should make length-converted catch curves particularly useful for stock assessment.

The following account shall be limited to animals whose mortality ( $Z$ ) can be described by the equation

$$N_t = N_0 e^{-Zt} \quad \dots 1)$$

where  $N_0$  is the initial number and  $N_t$  is the number of animals surviving after time  $t$ ; also, this account shall be limited to animals whose growth can be described by the von Bertalanffy Function (VBGF), which has the form

$$L_t = L_{\infty} (1 - e^{-K(t-t_0)}) \quad \dots 2)$$

where  $L_{\infty}$  is the asymptotic length,  $K$  a constant and  $t_0$  is the "age" the animal would have at length zero if they

had always grown in the manner described by the equation (as will be seen below,  $t_0$  is not needed at all in length-converted catch curves).

Age structured catch curves essentially consist of a plot of the natural logarithm of the number of fish in various age groups ( $N$ ) against their corresponding age ( $t$ ), or

$$\log_e N = a + bt \quad \dots 3)$$

$Z$  being estimated from the slope  $b$ , (with sign changed) of the descending, right arm of the plot.

The following assumptions are involved in this procedure:

- $Z$  is the same in all age groups included in the plot,
- all age groups used in the plot were recruited with the same abundance (or recruitment fluctuations have been small and of random character),
- all age groups used for the computation of  $Z$  are equally vulnerable to the gear used for sampling,
- the sample used is large enough and covers enough age groups to effectively represent the average population structure over the time considered. (See Beverton and Holt 1956, Ricker 1975, Chapter 2)

A major disadvantage of the age-structured catch curves represented by equation (3) is that they cannot be used in conjunction with animals that cannot be aged individually (e.g. shrimps, lobsters, some molluscs).

"Length-converted catch curves", as will be shown below, allow for the use of catch curves with such animals: moreover, the method, being based solely on length-frequency samples, allows for the use of very large samples without cons-

a) ICLARM Contribution No. 173.

truction of age-length keys.

The estimation of Z from a length-converted catch curve involves the following steps:

- pooling of individual length frequency samples into a single, large length-frequency sample representative of the population for the period under consideration (normally 1 year).
- construction of the catch curve proper, using the large sample above and a set of growth parameters (see below),
- estimation of Z from the descending right arm of the catch curve.

Pooling of length-frequency samples (e.g., of monthly samples) over a longer period of time (at least one year) is particularly needed in short-lived fish and shrimps, because their whole population structure is affected by seasonal "pulses" of recruitment (generally one or two per year). Also, to prevent a single larger (monthly) sample from unduly affecting the total (annual) sample, the various samples may be given the same weight, by conversion to percent length-frequency samples, prior to adding to obtain a single, overall sample. [Numerous alternatives to a scheme where each sample is given the same weight are possible; for example, it might be more appropriate to weight the samples by the square root of their size when fishery catch is not known, or by the catch, when it is known (H. Lassen, pers. comm.). However, empirical studies concerning appropriate sample sizes and weighting factors for length-converted catch curves are still lacking. Table 1 is given here to suggest sample sizes which at present seem appropriate.]

There are various methods by which a length-converted catch curve may be constructed. Common to all, however, is that they must account for the fact that fish growth (in length) is not linear, but slows down as length increases. This slowing down has the effect that older size groups contain more age groups than younger size groups do. Or put differently: it takes large fishes longer to "leave" a certain size

group, they "pile-up" (Baranov, as cited in Ricker 1975) in the size classes pertaining to old, large, slow-growing fish. Correcting for this effect is rather straightforward, and four methods by which this can be achieved are presented here. A first approach, analogous to, but improved over some of those discussed in Ricker (1975, p. 33 and 60-64), consists of multiplying the number in each length class by the growth rate ( $dl_i/dt$ ) of the fish in that class. This results in a catch curve equation of the form

$$\log_e [N_i (dl_i/dt)] = a + bt'_i \quad \dots 4)$$

where  $dl_i/dt$  is the growth rate and  $t'_i$  the relative age corresponding to length class  $i$ , respectively. In practice ( $dl_i/dt$ ) can be estimated from the VBGF as the growth rate pertaining to the median length, or "midlength" ( $L_i$ ) of length class  $i$ , while  $t'_i$  is estimated as the relative age corresponding to the midlength of class  $i$ , as estimated from

$$t' = [ -\log_e (1 - (L_i/L_\infty)) ] / K \quad \dots 5)$$

"Relative" ages are used here because using  $t_0$  (which leads to absolute ages) is not necessary in conjunction with catch curves, where Z is estimated from a slope.

A useful property of equation (4) is that it allows for readily estimating the bias caused by not accounting for the "pile-up-effect" mentioned above. This is done by first rewriting equation (4) as

$$\log_e N = a + bt'_i - \log_e (dl/dt) \quad \dots 6)$$

Now, in terms of the VBGF, the growth rate can be expressed as

$$dl/dt = \log_e (K L_\infty) + K(t' - t_0) \quad \dots 7)$$

where  $K$ ,  $L_\infty$  and  $t_0$  are parameters of the VBGF, and  $t'$  is the age corresponding to a given midlength. Inserting (7) into (6) and rearranging gives

where  $L_1$  and  $L_2$  are the lower and upper limit of length class  $i$ , respectively.

Thus, equation (11) accounts for the pile-up effect through division of the  $N_i$  values by  $\Delta t_i$ , the inverse of the growth rates by which the  $N_i$  values are multiplied in equation (4). Hence equation (11) is a slightly modified version of (4), and its properties e.g., with regard to not accounting for the piling-up effect are the same. Equation (11) was discussed previously in Pauly (1982, 1983) and Gulland (1983). Fig. 1 gives an example of such catch curve, based on the data in Table 2.

The criteria for the selection of the points to be included in the estimation of  $Z$  are

- as in age structured catch curve, the points belonging to the ascending, left arm of the curve must not be included because they represent incompletely selected and/or incompletely recruited animals, therefore the first point to be included ( $P_1$  in fig. 1) should generally be the point imme-

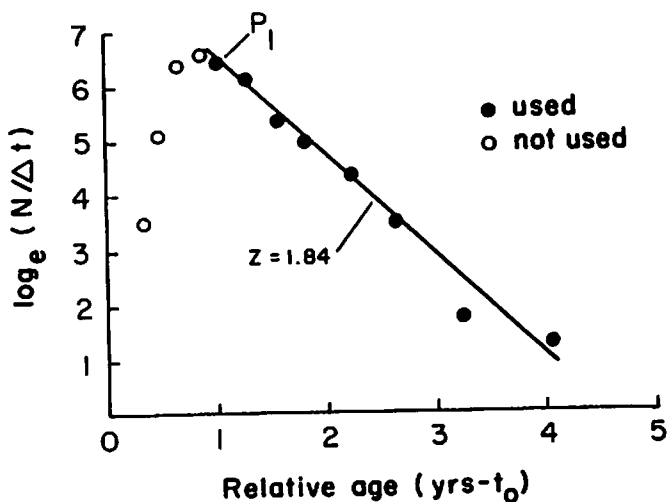


Fig. 1. Length-converted catch curve for banded grouper in the Visayan Sea, Philippines (based on data in Table 2).

diately to the right of the highest point (which generally will still be affected by incomplete selection and/or recruitment), points should be deleted that were obtained through conversion

Table 2 Data for the estimation of  $Z$  in the banded grouper (*Epinephelus sexfasciatus*) from the Visayan Sea, Philippines, using a length-converted catch curve (to be used with  $L_{\infty} = 30.9$  and  $K = 0.51$ , from Pauly, in press)

Midlength	$N^a$	$t$	$\log_e(N/\Delta t)$	Remarks	$t'$
5 cm	5	0.151	3.500	not used, ascending part of curve	0.346
7 cm	29	0.164	5.175		0.504
9 cm	114	0.180	6.451		0.675
11 cm	161	0.197	6.706		0.863
13 cm	143	0.213	6.509	used, descending straight part of curve	1.07
15 cm	118	0.247	6.169		1.30
17 cm	61	0.283	5.373		1.57
19 cm	50	0.330	5.021		1.87
21 cm	32	0.397	4.39		2.23
23 cm	17	0.498	3.530		2.67
25 cm	4	0.670	1.787		3.25
27 cm	4	1.028	1.359		4.06

a) obtained by summing up samples from various months.

**Table 1. Criteria for assessing the suitability of length-frequency samples for estimating Z (modified from Munro 1982). It is assumed that the samples cover the whole length range of the fish, gear selection is accounted for and that the sizes of the individual samples are more or less equal.**

Total sample size (# of fish)	Time period (in months) over which data for individual samples were accumulated				
	12	6	3	2	1
1 - 100	0	0	0	0	0
101 - 500	0	0	1	2	2
501 - 1000	1	1	2	3	4
1001 - 1500	1	2	3	4	5
> 1500	2	3	4	5	5+

0 = not usable                      2 = fair                              4 = very good  
 1 = poor                              3 = good                              5 = excellent

$$\log_e N = a + bt' - \log_e KL_\infty - Kt' + Kt_0 \quad \dots 8)$$

This equation, it will be noted, has 3 constant terms with regard to the variables  $N$  and  $t'$ , namely  $a$ ,  $\log_e(KL_\infty)$  and  $Kt_0$ . Since  $Z$  in equation (4) is estimated as a slope, these 3 constant terms can be grouped into one single new term ( $a'$ ) which becomes the intercept of a new equation for a length-converted catch curve, i.e.

$$\log_e N = a' + (-K + b) t' \quad \dots 9)$$

Therefore, total mortality ( $Z$ ) can be estimated from the slope - with sign changed - of equation (9) plus  $K$ , or

$$-b + K = Z \quad \dots 10)$$

It follows from this that the bias resulting from the non-consideration of the pile-up effect (i.e., resulting from using  $\log_e N$  instead of  $\log_e(N \, dl/dt^a)$  as ordinate of a length-converted catch curve) is equal to  $K$ .

Two practical applications of this finding come to mind:

- i - it becomes possible to correct a posteriori biased values of  $Z$  obtained by various authors who didn't account for the pile-

up effect by simply adding  $K$  to their (biased) estimate of  $Z$  (see e.g., Berry 1970, Nzioka 1983),

- ii- the estimation of  $Z$  from a length-converted catch curve becomes simpler, since one can first ignore the "pile-up" effect, then compensate for it by adding  $K$  to the absolute value of the curve's slope.

When  $K$  is not known, equations such as (4) and (9) can still be used; in such cases, a value of 1 (one) should be used instead of  $K$  in equation (5), used for computing the relative ages. The slope of the catch curve, with sign changed will then be equal to  $(Z/K) - 1$ .

Another type of length-converted catch curve is defined by the equation

$$\ln N_i / \Delta t_i = a + bt_i \quad \dots 11)$$

where  $N_i$  and  $t_i$  are defined as in equation (4), and where  $\Delta t_i$  is the time needed on the average by the fish to grow through length class  $i$  (Pauly 1982, 1983, Gulland 1983). The value of  $\Delta t_i$  is estimated from

$$\Delta t_i = (\log_e(L_\infty - L_1 / L_\infty - L_2)) / K \quad \dots 12)$$

from lengths within 5% of  $L_{\infty}$ , as their relative age will tend to be overestimated, and points based on a few fish only (e.g. less than 5 may be omitted when they do not fit along the straight part of the catch curve).

P. Sparre (pers. comm.) derived a length-converted catch curve which corrects for the non-linearity of equations (1) and (2) and for the fact that some mortality occurs within each length class and which has the form

$$\log_e(N_i / (1 - e^{-Z_j \Delta t_i})) = a - Z_{j+1} t_i^* \quad \dots(13)$$

where  $N_i$  and  $\Delta t_i$  are defined as above,  $t_i^*$  is the relative age corresponding to the lower limit of length class  $i$  and where  $Z_j$  and  $Z_{j+1}$  are initial, and improved estimates of  $Z$ , respectively.

Equation (13), which can be solved only iteratively (i.e. using a first guess of  $Z$ , then improving it successively), has the definite advantage that it was derived rigorously, and uses none of the approximations involved in the other types of length-converted catch curves.

Numerous comparisons of  $Z$  values obtained through equation (13) with estimates of  $Z$  obtained through equation (4) and (11) showed, however, that equations (4) and (11) usually underestimate  $Z$  by less than 1% when the available length-frequency data are arranged in a reasonably large number of classes (say  $> 10$ ). The most useful aspect of equation (12) is thus that it justifies the use of the simpler equations for most routine situations.

### References

- Berry, R.J. 1970. Shrimp mortality rates derived from fishery statistics. Proc. Gulf and Caribb. Fish. Inst. 22:66-78.
- Beverton, R.J.H. and S.J. Holt. 1956. A review of methods for estimating mortality rates in fish populations, with special reference to sources of bias in catch sampling.

- Rapp. P.-v. Reun. Cons. Perm. Inter. Explor. Mer 140:67-83.
- Gulland, J.A. 1983. Fish stock assessment: a manual of basic methods. FAO/Wiley Ser. on Food and Agriculture. Vol.1, 223 p.
- Jones, R. 1981. the use of length composition data in fish stock assessment (with notes on VPA and Cohort Analysis). FAO Fish. Circ. No. 734, 55 p.
- Munro, J.L. 1982. Estimation of biological and fishery parameters in coral reef fisheries p. 71-82. In D. Pauly and G.I. Murphy (eds) Theory and Management of Tropical Fisheries. ICLARM Conf. Proc. 9, 360 p.
- Nzioka, R.M. 1983. Biology of the smallspotted grunt Pomadasyse opercularis (Playfair 1866) (Pisces: Pomadasyidae) around Malindi in Kenya. Kenya J. Sci. Techn. (15):69-81.
- Pauly, D. 1982. Studying single-species dynamics in multispecies context, p. 33-70. In D. Pauly and G.I. Murphy (eds) Theory and Management of Tropical Fisheries. ICLARM Conf. Proc. 9, 360 p.
- Pauly, D. 1983. Some simple methods for the assessment of tropical fish stocks. FAO Fish Tech. Pap. No. 234, 52 p.
- Pauly, D. Fish population dynamics in tropical waters: a manual for use with programmable calculators. ICLARM Studies and Reviews 8. (in press)
- Ricker, W.E. 1975. Computation and interpretation of biological statistics of fish populations. Bull. Fish. Res. Board Can. (191), 382 p.

a) This manual, which was announced somewhat prematurely in the ICLARM Newsletter and elsewhere has been reviewed externally and completely revised. It is now definitely "in press" and will become available in the first half of 1984. This contribution was extracted from Chapter 5 of the final draft.