Beverton and Holt Equations: Spreadsheet Functions and Uncertainty

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Abstract

This contribution illustrates how modern spreadsheets aid the calculation and visualization of yield models and how the effects of uncertainties may be incorporated using Monte Carlo simulation. It is argued that analogous approaches can be implemented for other assessment models of simple to medium complexity—justifying wider use of spreadsheets in fisheries analysis and training.

Introduction

In 1948, Ray Beverton and Sydney Holt at the Lowestoft Fisheries Laboratory, U.K., used one of the first electro-mechanical calculators to speed their work on yield-per-recruit analyses. They also employed a 3-D paper sculpture to help visualize the yield-per-recruit surface. A photograph of them using both of these aids was published as the frontpiece of the 1993 reprint of the classic book on the dynamics of exploited fish populations (Beverton and Holt 1957). Despite such aids, calculation of the yield equations remained a fatiguing and error prone task, carried out in Beverton and Holt's day by "experienced computers"—people who were able to stand the tedium and concentration required. In the 1960's, FAO published booklets of proformas (Gulland 1969) to help store intermediate values of the multi-part equations. Even recent guides to the calculations (Sparre and Venema 1992) provide associated pre-programmed software rather than guiding trainees to do it themselves, as the latter is potentially error prone (Hilborn 1996).

Since the end of the 1980s, the use of spreadsheets has facilitated both training and analysis in fisheries by allowing researchers to tailor models and outputs to each circumstance. This brief paper aims not only to show how modern spreadsheets aid the calculation and visualization of yield models (which has been known for some time), but also how the effects of uncertainties and risk may be incorporated through Monte Carlo simulation with the rather rapid improvement of programming languages like Microsoft Visual Basic inside Microsoft Excel spreadsheets (Pitcher and Hart 1981; Restrepo and Fox 1988). Moreover, despite the advent of multi-parameter complex models based on the stock synthesis approach (Methot 1990), simple equilibrium yield-per-recruit and biomass-per-recruit analyses remain valuable first steps in fisheries assessment in data sparse fisheries throughout the world. Although demonstrated here with the classic Beverton and Holt equation, these advantages apply as much to other forms of yield equations, such as the length-based analysis based on the Thompson and Bell equations (Sparre and Venema 1992) and also to non-equilibrium analyses. For example, multispecies yield-per-recruit and value-per-recruit analyses (Murawski 1984) can be programmed in Microsoft Visual Basic within a Microsoft Excel spreadsheet and uncertainties evaluated in a very flexible way compared to pre-programmed software (Sparre and Willman 1992).

Spreadsheet Functions and Monte Carlo Simulations

As an example, Fig. 1 shows graphs of three of the principal Beverton and Holt assessment equations, yield-per-recruit (YPR), biomass-per-recruit (BPR) and mean weight (MW) of fish in the catch calculated using the Microsoft Visual Basic functions given in Appendix 1. These are plotted as 3-dimensional surfaces against fishing mortality rate (F) and age of entry (and hence mesh size) t_c. In addition, the slope of the yield-per-recruit surface is plotted to illustrate the position of the $F_{0.1}$ line. $F_{0.1}$ was introduced by Gulland (1968) as an arbitrary but consistent reference point below Y_{max}, expressing decreasing returns with further increase in effort and calculated as the point at which the slope of YPR is 0.1 of the slope at the origin, for the same t_c.

Such pictorial representations make the consequences of the surface shapes clear. For example, in YPR we can clearly see the promontory of the surface at lower t_c values where Y_{max} occurs at ever

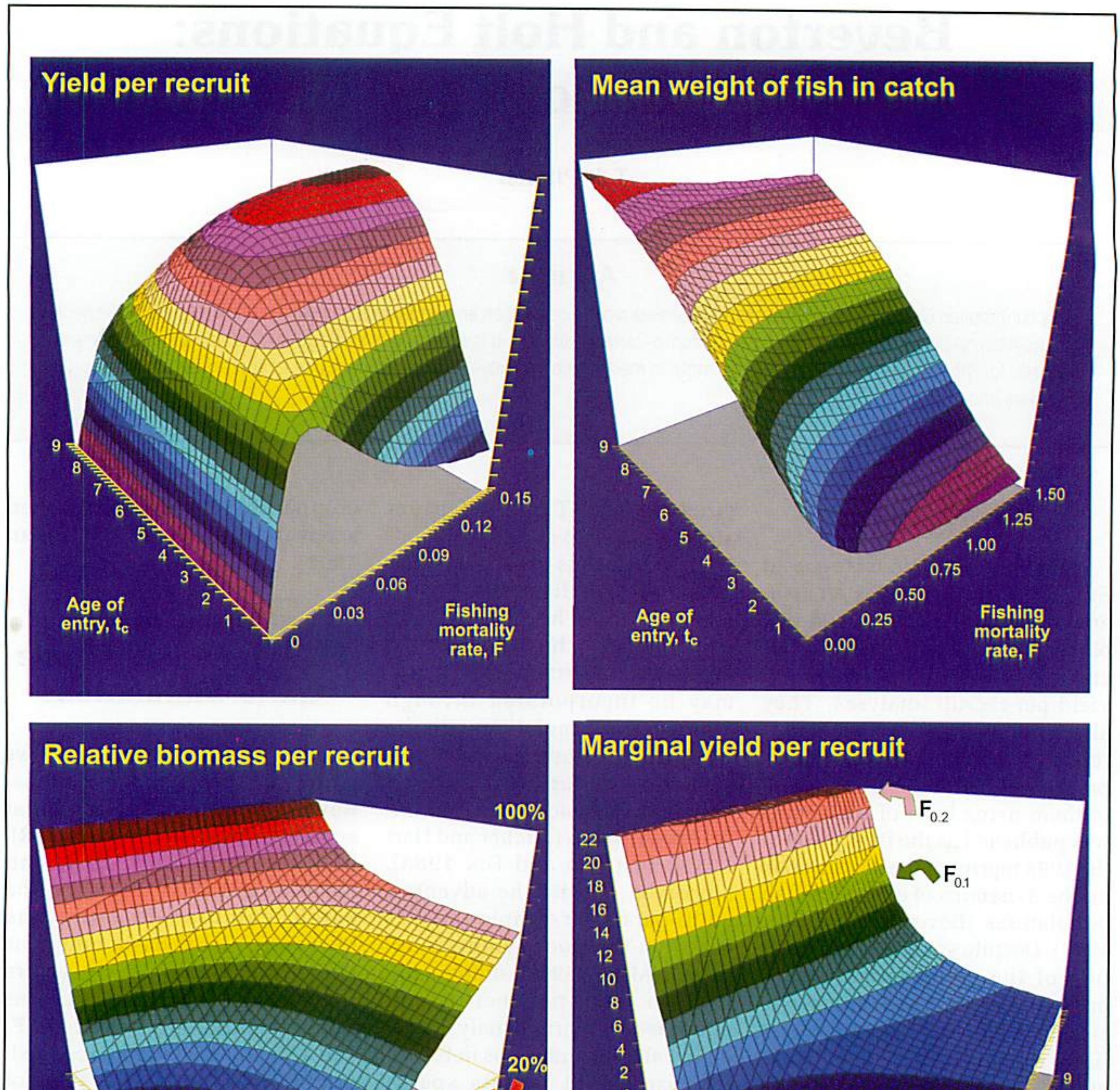

Fig. 1. Three-dimensional surface plots of Beverton and Holt fishery assessments.
Clockwise from top left: yield-per-recruit (YPR), *mean weight of fish in catch* (MW), *marginal yield-per-recruit and relative biomass-per-recruit* (BPR), *calculated using the Microsoft Visual Basic functions listed in Appendix 1.* YPR: *contours are drawn at 5% of* Y_{max}; MW: *contours are drawn at 5% of maximum in unexploited population. Marginal* YPR: *approximate slopes obtained by interpolating adjacent cells of the* YPR *surface (hence different F axis labels), zero plane indicted.* BPR: *contours drawn at 5% intervals; 20% of unexploited biomass plane indicated. Model parameter values* W_∞ = *3 000;* K = *0.25;* t_0 = *-0.2;* t_r =*0.75;* t_l = *20;* M = *0.22.*

higher F as t_c increases. The peak of the YPR surface, Y_{max} at t_{opt} is also seen at high fishing mortality (the peak is actually at infinity). For **BPR**, we can see that the line of 20% of unexploited biomass occurs at approximately the same age of entry (t_c = 5 + year) as the peak of the yield per recruit surface (Y_{max}). The 20% reference value has been widely used in fisheries assessment since Beddington and Cooke (1983) (Francis 1993). In the marginal **YPR** plot, the more conservative $F_{0.2}$ line is shown. On the **MW** plot, we can see that even at $F_{0.1}$ fish half the average size of those in the unexploited population will be caught.

Estimation uncertainties for various reference points on the yield surfaces may be estimated through a series of Monte Carlo simulations. Each simulation comprises an estimate of the output value (say $F_{0.1}$) made by choosing values randomly from the error distributions attached to each input parameter of the model. After a large number of simulations are made (often 1 000), the mean and distribution of the output results may be analyzed. This method is equivalent to the unconditional parametric bootstrap procedure outlined by Smith et al. (1993). Simulations were implemented here using proprietary add-on software for the Microsoft Excel spreadsheet (Decisioneering Inc., 1515 Arapahoe St., Denver, Co. USA, 80202: www.decisioneering.com). Although similar procedures could be self-programmed in Microsoft Visual Basic, the advantage of the proprietary add-on, apart from avoiding much debugging of one's own routines, is that many different error distributions (such as gamma, log Normal, uniform or triangular between upper and lower bounds, and user-defined) can be set up independently for each input parameter. Distributions can be fitted to data. Moreover, correlations between parameters, such as the strong negative correlation between the von Bertalanffy growth rate and asymptotic size, can be set up to constrain the choice of random numbers. Microsoft Visual Basic calls are also available for more complex models that may include optimization routines, calls to other external procedures, or employed in user-written Microsoft Visual Basic programs. Output options include analysis of distributions,

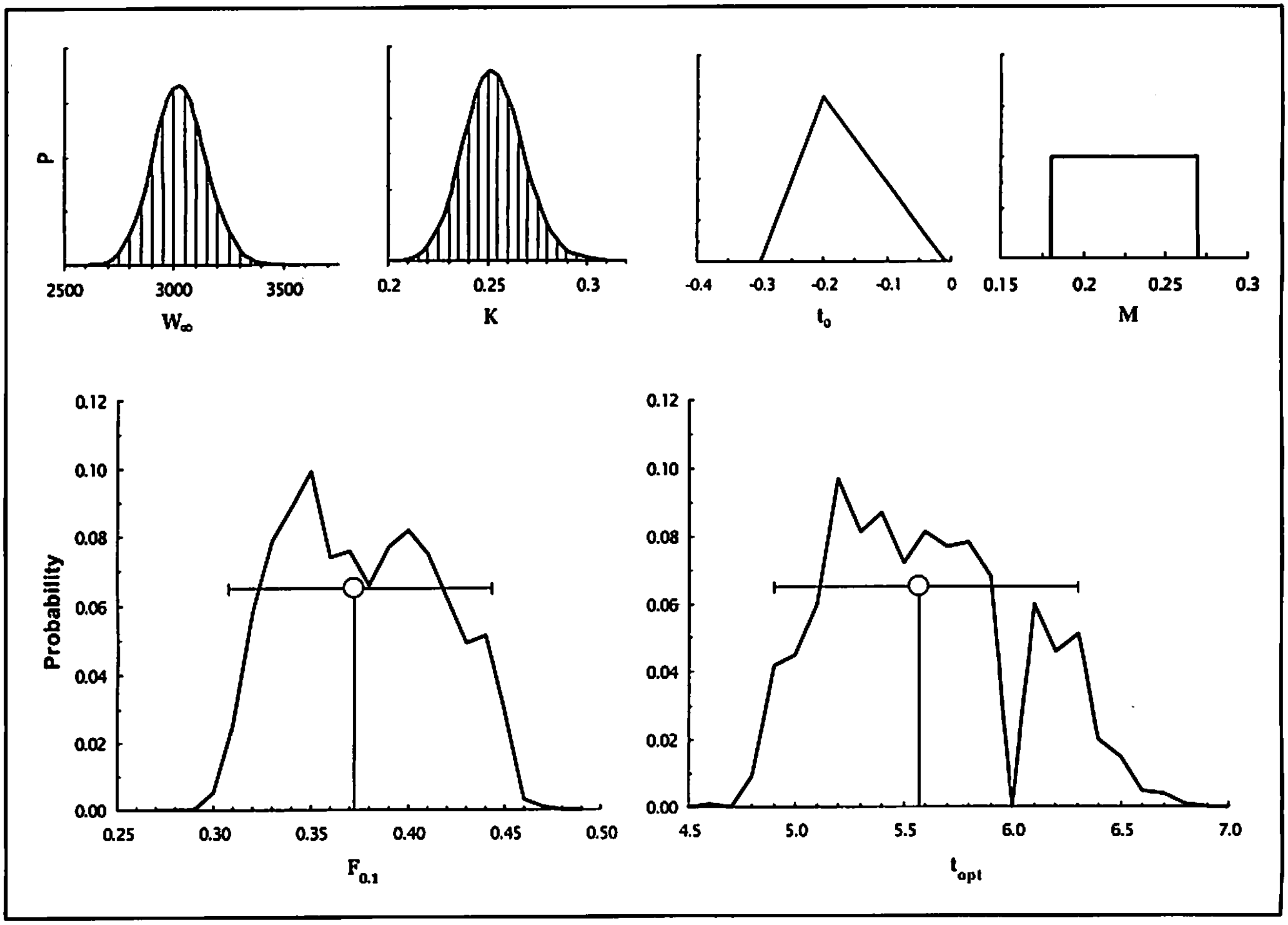

Fig. 2. Error distributions attached to four yield-per-recruit model input parameters (W_∞, K, t_0 and M: top row) and error distributions on two model output biological reference points ($F_{0.1}$ and t_{opt}: bottom row) after 1 000 Monte Carlo simulations of the bootstrap procedure. Vertical axis for top row is relative probability, for bottom row is probability. For outputs, mean and 95% percentiles are indicated by bars.

Appendix 1. Microsoft Visual Basic functions for use in Microsoft Excel for the Beverton and Holt equations.

Each function may be called by a spreadsheet cell for values of age of entry (t_c) and fishing mortality (F), given parameters values for W_∞ = asymptotic weight; K = von Bertalanffy growth rate; t_0 = von Bertalanffy model start time; M = natural mortality rate; t_r = age of recruitment to fishing grounds; t_l = highest age in unfished stock. Copies of the spreadsheets used may be downloaded from the web site: http:\\fisheries.com

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Function ypr(winf, k, t0, m, f, tr, tc, tl)
Dim u(4)
u(1) = 1: u(2) = -3: u(3) = 3: u(4) = -1
Sum = 0
For i = 0 To 3
a = u(i + 1) / (f + m + i * k)
b = Exp(-i * k * (tc - t0))
c = 1 - Exp(-(f + m + i * k) * (tl - tc))
Sum = Sum + a * b * c
Next i
ypr = Sum * winf * f * Exp(-m * (tc - tr))
End Function

Function bpr(winf, k, t0, m, f, tr, tc, tl)
Dim u(4)
u(1) = 1: u(2) = -3: u(3) = 3: u(4) = -1
Sum = 0
For i = 0 To 3
a = u(i + 1) * Exp(-i * k * (tr - t0))
b = (1 - Exp(-((m + (i * k)) * (tc - tr)))) / (m + (i * k))
c = (Exp(-((m + (i * k)) * (tc - tr)))) * (1 - Exp(-((f + m + i * k) * (tl - tc))))
d = f + m + (i * k)
Sum = Sum + a * (b + (c / d))
Next i
bpr = Sum * winf
End Function

Function mwt(winf, k, t0, m, f, tr, tc, tl)
Dim u(4)
u(1) = 1: u(2) = -3: u(3) = 3: u(4) = -1
Sum = 0
For i = 0 To 3
a = u(i + 1) / (f + m + i * k)
b = Exp(-i * k * (tc - t0))
c = 1 - Exp(-(f + m + i * k) * (tl - tc))
Sum = Sum + a * b * c
Next i
v = Exp(-(f + m) * (tl - tc))
mwt = Sum * winf * (f + m) / (1 - v)
End Function

Function mage(winf, k, t0, m, f, tr, tc, tl)
v = Exp(-(f + m) * (tl - tc))
mage = 1 / (f + m) + (tc - tl * v) / (1 - v)
End Function
```

graphical output and storing of results of all simulations for analysis by the user.

Fig. 2 shows the results from an analysis set up to examine uncertainty in $F_{0.1}$ and t_{opt} for the yield-per-recruit model illustrated in Fig. 1. Error distributions on the main four input parameters for yield-per-recruit, W_T, M, K and t_0, are set up to illustrate the range of options. Final error distributions on t_{opt} and $F_{0.1}$ resulting from 1 000 Monte Carlo bootstrap simulations are also illustrated. The error distributions are strongly platykurtic and, hence, are represented by the mean and the 95th percentile. $F_{0.1}$ was approximated on the spreadsheet by setting up a vector of **YPR** values with increasing **F** at the desired t_c, comparing the calculated slopes on this vector with the slope at very low **F**, and obtaining $F_{0.1}$ by interpolation between adjacent **F** values when the values matched. Smaller intervals between **F** values in the spreadsheet vector can increase the accuracy of this procedure. A similar procedure was adopted to estimate t_{opt} from a vector of **YPR** values with increasing t_c at very high **F** (i.e., the right side of the **YPR** surface in Fig. 1).

Conclusion

Analogous methods can be easily adapted to many fisheries assessment models of simple to medium complexity that can be implemented on spreadsheets, e.g., many of the models used in ICES (1997). Fisheries scientists easily forget that complexity for its own sake is not always helpful. Simpler models often perform stock assessment tasks better (Walters 1996).

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Length-Weight Relationship of Fishes from Yemen Waters (Gulf of Aden and Red Sea)

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The data for this study were gathered between 1993 and 1996 on board commercial trawlers from Somalia, China and Yemen and also from the research vessel *Ibn Magid* belonging to the Marine Science and Resources Research Centre, Aden, Republic of Yemen.

Fish were identified using FAO species identification literature (Fischer and Bianchi 1984). All fish were measured to the nearest mm (total length) and weighed to the nearest g. Sex was determined by dissection after the length and weight had been measured. The length-weight relationships were calculated using least-squares regression on log-transformed data and the parameters of the relationship of the form of $W = aL^b$ are summarized in Table 1. Maximum and minimum sizes of fish sampled are also given. Common names and recent changes in nomenclature were taken from FishBase (Froese and Pauly 1996).